CAMBRIDGE
INTERNATIONAL EXAMINATIONS

NOVEMBER 2002

GCE Advanced Level
GCE Advanced Subsidiary Level

MARK SCHEME

MAXIMUM MARK : 50

SYLLABUS/COMPONENT :9709 /5, 8719 /5

MATHEMATICS
(Mechanics 2)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1		$r = 4$cm	B1	
		Uses $v = wr$	M1	
		Speed is 20cm s^{-1} (FT if $r = \frac{1}{3} \times$ candidates perp. distance from B)	A1	3

2	(i)	Takes moments about B [$T\cos60^\circ \times 2 = 10g \times 1$]	M1	
		Obtains tension as 100 N	A1	2
	(ii)	Uses Hooke's Law (for expression in x or l only)	M1	
		Obtains $100 = 200(3 - L)/L$ or $100 = 200x/(3 - x)$	A1 ft	
		Obtains natural length as 2 m	A1	3

3	(i)	$x = 10t$, $y = -5t^2$	B1	
		Eliminates t to find an equation in x and y (allow if candidate derives the general trajectory equation)	M1	
		Obtains $y = -x^2/20$ (Allow B1/3 for putting $\theta = 0$ and $V = 10$ in traj. equation)	A1	3
	(ii)	Uses $\tan\theta = dy/dx$ or $\tan\theta = \dot{y}/\dot{x}$	M1	
		Obtains $x = 30$ when $y = -45$, or $t = 3$ when $y = -45$, or $\dot{x} = 10$ and $\dot{y} = (\pm)30$	A1	
		Obtains angle as 108.4° (108.435) or 71.6° (71.565)	A1	3

4		$a = 4^2/0.8$ [= 20]	B1	
		Uses Newton's 2nd law horizontally to obtain a 3 term equation	M1	
		Obtains $(T_P + T_Q)\cos30^\circ = 0.5 \times 20$ [$T_P + T_Q = \frac{20}{\sqrt{3}}$]	A1 ft	
		Resolves forces vertically to obtain a 3 term equation	M1	
		Obtains $T_P\cos60^\circ = T_Q\cos60^\circ + 5$ [$T_P - T_Q = 10$]	A1	
Alternatively for the above 4 marks Uses Newton's 2nd law perpendicular to BQ to obtain a 3 term equation M2 Obtains $T_P\cos30^\circ - 0.5g\cos30^\circ = 0.5 \times 20\cos60^\circ$ [$T_P = 5 + \frac{10}{\sqrt{3}}$] A2 ft [SR Allow A1 with 1 sign or trigonometric error]				
		Obtains tension in PB as 10.8 N (10.7735)	A1	6

NB Use of equal tensions can score B1, M1, A0, M1, A0 at most.

5	(i)	GPE = $0.075g(d \sin 30°)$ or $0.075g(d+x)\sin 30°$	B1	
		EPE = $1.5(d-2)^2/2\times2$ or $1.5x^2/2\times2$	B1	
		Uses the principle of conservation of energy to form an equation with GPE and EPE terms $[\frac{3}{8}d = \frac{3}{8}(d-2)^2$ or $\frac{3}{8}(2+x) = \frac{3}{8}x^2]$	M1*	
		Attempts to solve a quadratic equation in d $[(d-1)(d-4)=0]$ or attempts to solve a quadratic equation in x and uses $d = x + 2$ $[(x+1)(x-2)=0$ and $d = 2 + 2]$	M1 dep	
		Obtains distance as 4m	A1	5
	(ii)	Obtains the tension at the lowest point as 1.5 N ft for $1.5(d-2)/2$	B1 ft	
		Uses Newton's 2nd law to obtain a 3 term equation	M1	
		Obtains $1.5 - 0.075g \sin 30° = 0.075a$	A1 ft	
		Obtains acceleration as $15 ms^{-2}$	A1	4

6	(i)	Uses Newton's 2nd law and $a = v\,dv/dx$, and attempts to integrate $[(1/10)\,v\,dv/dx = -v/200]$	M1*	
		$v = -x/20 \quad (+C)$	A1	
		Uses $v(0) = 5$ to find C	M1 dep	
		Obtains $v = -x/20 + 5$ (a.e.f.)	A1	4
	(ii)	Uses $v = dx/dt$, separates the variables and integrates $[\int \frac{1}{100-x}dx = \int \frac{1}{20}dt]$	M1*	
		Obtains $\ln(100-x) = -t/20 (+ C)$	A1	
		Uses $x = 0$ when $t = 0$ to obtain $t = 20[\ln 100 - \ln(100-x)]$ ft only if the term in x is logarithmic	A1 ft	
		For taking anti-logarithms throughout the equation $[100 - x = 100e^{-t/20}]$ NB $\ln(100-x) = -t/20 + C \rightarrow 100-x = e^{-t/20} + e^{C}$ is M0	M1 dep	
		Obtains $x = 100(1 - e^{-t/20})$ (a.e.f.)	A1	5
Alternatively for the above 9 marks Uses Newton's 2nd law with $a = dv/dt$, separates the variables and integrates $[\int \frac{1}{v}dv = -\int \frac{1}{20}dt]$ — M1* Obtains $\ln v = -t/20 \quad (+C)$ — A1 Uses $v = 5$ when $t = 0$ to obtain $t = 20[\ln 5 - \ln v]$ ft only if the term in v is logarithmic — A1ft For taking anti-logarithms throughout the equation $[v = 5e^{-t/20}]$ — M1 dep Uses $v = dx/dt$ and integrates $[x = \int 5e^{-t/20}dt]$ — M1* Obtains $x = -100e^{-t/20}$ (+C) — A1 Uses $x = 0$ when $t = 0$ to obtain $x = 100(1 - e^{-t/20})$ — A1 Eliminates the exponential term from $x = 100(1 - e^{-t/20})$ and $v = 5e^{-t/20}$ to obtain an equation in x and v $[x = 100(1 - v/5)]$ — M1 dep Obtains $v = -x/20 + 5$ — A1				
	(iii)	$x = 100(1 - e^{-t/20})$ and $e^{-t/20}$ is +ve for all $t \rightarrow x < 100$	B1	1

NB If (i) is solved as in scheme and then (ii) is solved using the alternative method, the 5 marks awarded for (ii) from the alternative method are M1* (A0), A1 (not FT), M1 (dep), M1* (uses $v = \frac{dx}{dt}$ and integrate or subst. for v from (i)), (A0) A1 (M0 dep) (A0).

7	(i)	Uses $(A_1 \pm A_2)x = A_1x_1 \pm A_2x_2$ to find x [(25x5 + 15x5)x = 25x5x12.5 + 15x5x2.5]	M1	
		Obtains x = 8.75	A1	
		Uses $(A_1 \pm A_2)y = A_1y_1 \pm A_2y_2$ to find y [(25x5 + 15x5)y = 25x5x2.5 + 15x5x12.5]	M1	
		Obtains y = 6.25	A1	4
	(ii)	States or obtains $\mu = \tan\alpha$ for prism on point of sliding	B1	
		States or obtains $\tan\alpha \leqslant x/y$ for prism not toppled	M1	
		~~Eliminates tan α from μ = tan α and tan $\alpha \leqslant x/y$, and substitutes for x and y~~ [~~μ < 8.75/6.25~~] obtains $\tan\alpha \leqslant 8.75/6.25$	A1	
		Coefficient of friction is less than 7/5 (convincing explanation for inequality	A1	4
	(iii)	States or obtains $\tan\beta = y/x$ for prism on point of toppling	M1	
		States or obtains $\mu \geqslant \tan\beta$ for prism not sliding (or on the point of sliding)	B1	
		Eliminates tan β from $\tan\beta = y/x$ and $\mu > \tan\beta$, and substitutes for x and y [μ > 6.25/8.75] to obtain the least value of the coefficient of friction as 5/7	A1	3

(convincing explanation for inequality)